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International Journal of Heat and Mass Transfer 44 (2001) 2787–2789

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Letter to the Editors

Comments on "A fractal geometry model for evaluating permeabilities of porous preforms used in liquid composite molding"

In the recently published paper by Pitchumani and Ramakrishnan [1], the authors presented a fractal geometry model for evaluating permeabilities of porous preforms used in liquid composite molding. However, the model is highly questionable. Some comments are as the follows:

1. Eqs. (6)–(10) in the paper [1] lack the sound mathematical base.

According to Eqs. (2)–(5) in the paper [1], the definition of Q (page 2219): "total flow rate through the preform", and the further detailed definition of Q as (page 2223) "Further, the total volumetric flowrate, Q, through the representative preform volume is obtained by integrating the flowrate contribution by the pores in every dλ interval, over the entire range of prevalent pore sizes, $\lambda_{\min}$ to $\lambda_{\max}$ in the preform." The corresponding equations should be

$$Q = \int_{\lambda_{\min}}^{\lambda_{\max}} q(\lambda)\,\mathrm{d}N(\lambda) = g_q \frac{\Delta p}{\mu} \lambda_{\max}^3 \beta^{d_T} \left[d_N \frac{(1-\alpha^{3+d_T-d_N})}{3+d_T-d_N} \right], \quad \text{(C1)}$$

$$k = \frac{\mu L_0 Q}{\Delta p A_0} = g_q \lambda_{\max}^2 \beta^{1+d_T} \left[d_N \frac{(1-\alpha^{3+d_T-d_N})}{3+d_T-d_N} \right], \quad \text{(C2)}$$

$$v_p = 1 - v_f = \frac{1}{L_0^2} \int_{\lambda_{\min}}^{\lambda_{\max}} g_v \lambda^2\,\mathrm{d}N(\lambda) = g_v \beta^2 \left[d_N \left(\frac{1-\alpha^{2-d_N}}{2-d_N} \right) \right], \quad \text{(C3)}$$

$$\beta = \sqrt{\left(\frac{2-d_N}{d_N} \right)}, \quad \text{(C4)}$$

$$\alpha = \left(1 - \frac{1-v_f}{g_v} \right)^{\frac{1}{2-d_N}}. \quad \text{(C5)}$$

Eqs. (C1)–(C3) are the mathematically integrated results (see Appendix A for the detail). But this does not imply that Eqs. (C1)–(C3) are the correct fractal model for permeabilities. Eqs. (6)–(8) in the paper [1] can be obtained *only* by adding additional terms $g_q(\Delta P/\mu)\lambda_{\max}^3\beta^{d_T}$, $g_q\lambda_{\max}^2\beta^{d_T+1}$ and $g_v\beta^2$ into the right side of Eqs. (C1)–(C3), respectively, and this means $Q + g_q(\Delta P/\mu)\lambda_{\max}^3\beta^{d_T} = Q$, $k + g_q\lambda_{\max}^2\beta^{d_T+1} = k$ and $v_p + g_v\beta^2 = v_p$. From the definition of Q, Eqs. (C1)–(C3) should be the final results, therefore, there should be $Q + g_q(\Delta P/\mu)\lambda_{\max}^3\beta^{d_T} \neq Q$, so as to $k + g_q\lambda_{\max}^2\beta^{d_T+1} \neq k$ and $v_p + g_v\beta^2 \neq v_p$, and thus Eqs. (6)–(8) are mathematically incorrect.

2. When the pore area fractal dimension $d_N = 2$, $\beta = 0$ from Eq. (9) and the model predicted permeability $k = 0$ from Eq. (7), and Fig. 7 illustrates the results of the model. The paper [1] also states (page 2228) that "An interesting fact elucidated in Fig. 7(a) is that for any tortuosity dimension, as the area dimension, d_N, approaches its largest possible value of 2, the permeability approaches zero." However, when $d_N = 2$, $\beta = 0$ from Eq. (9) and $v_p = 0$ (or $v_f = 1$) from Eq. (8), this will result in $\alpha = 1$ from Eq. (10). $\alpha = 1$ corresponds to the physical situation of the preform consisting of pores with the size of $\lambda_{\min} = \lambda_{\max}$, and this will lead to the non-zero flowrate ($Q \neq 0$) and non-zero permeability ($k \neq 0$). Therefore, Eqs. (7)–(10) are contradictory each other.

As a result, the model proposed by Pitchumani and Ramakrishnan [1] is an erroneous one.

Eq. (3) (page 2222) in the paper [1] defines that "where d_N , the pore area dimension, is the fractal dimension of the intersecting pore cross-sections with a plane normal to the flow direction. Since d_N defines a fractal surface in a two-dimensional plane, its value lies in the range $1 < d_N < 2$." The paper [1] also states (page 2228) that "An interesting fact elucidated in Fig. 7(a) is that for any tortuosity dimension, as the area dimension, d_N, approaches its largest possible value of 2, the permeability approaches zero." Eqs. (6)–(8) DO yield the zero flow rate ($Q = 0$), zero permeability ($k = 0$) and zero pore volume fraction ($v_p = 0$) as $d_N = 2$ (because $d_N = 2$, $\beta = 0$ from Eq. (9), $Q = 0$ from Eq. (6), $k = 0$ from Eq. (7) and $v_p = 0$ from Eq. (8)).

Actually, according to the definition on the "box counting method" (for evaluating the pore area dimension d_N given by the authors [1], page 2225), "$N(L)$ now pertains to the number of boxes required to completely cover the pore areas." This will lead to $d_N = 2$ when a representative cross-section completely consists of pores or is only one pore, which means that the *pore* volume fraction v_p of this cross-section is 1. A cross-section with

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$v_p = 1$ will have the largest permeability for flow through this cross-section. Obviously, Eqs. (6)–(8) can be proved to be incorrect by themselves.

I here would like to give the following examples/references to further provide evidences.

Example 1: Skjeltorp [2], after using the "box counting method" for his experimental results, pointed out that (page 319) "It is also of interest to find D for different *packing* fractions (or coverage) defined as $\eta =$ *packed* area/total area… As expected, D approaches 2 as η increases toward 1 (*compact* structure with no holes)." In this paper [2], D is (page 320): "The fractal dimension of the *packed* regions." Compared with the paper [2], the *pore* in the paper [1] is corresponding to the *packed* in the paper [2]. Thus, the *pore* area fractal dimension d_N in the paper [1] will approach 2 as the *pore* volume fraction v_p(= *pore* area/total area) increases toward 1 (*porous* structure with no impermeable substance), and this ($v_p = 1$) should result in the maximum permeability. This conclusion is exactly opposite to the model presented by Pitchumani and Ramakrishnan [1].

Example 2: Let us examine an another example, a generator of Sierpinski Carpet as shown in Fig. 1 (of the present paper), the *pore* area fractal dimension is $d_N = 1.8928$ [3] and the *pore* volume fraction is $v_p = 8/9$. If removing the central impermeable (black) substance from Fig. 1 (of the present paper), we have a square pore with the *pore* fractal dimension $d_N = 2$ and the *pore* volume fraction $v_p = 1$ as shown in Fig. 2 (of the present paper). It can be clearly seen that the permeability value of Fig. 2 (of the present paper) with $v_p = 1$ and $d_N = 2$ is higher than that of Fig. 1 (of the present paper) with $v_p = 8/9$ and $d_N = 1.8928$. In fact, the cross-section, as shown in Fig. 2 (of the present paper) with $v_p = 1$ and with $d_N = 2$, has the largest permeability compared with other Sierpinski Carpets with $v_p < 1$ and $d_N < 2$ in two-dimensions.

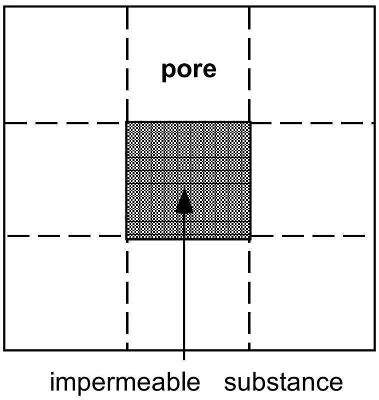

Fig. 1. A generator of Sierpinski Carpet with the pore fractal dimension $D_f = 1.8928$ and $v_p = 8/9$ and it has the lower permeability value than that of Fig. 2.

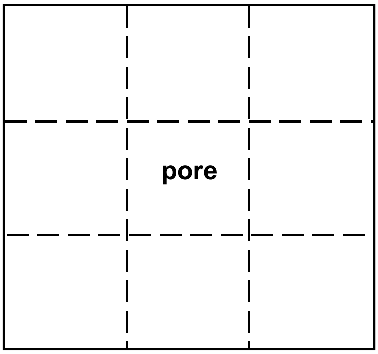

Fig. 2. A square pore with the pore fractal dimension $D_f = 2$ and $v_p = 1$ and it has the higher permeability value than that of Fig. 1.

In addition to these two examples, some similar results and evidences can be found in many literatures [3–8].

Appendix A

From Eqs. (4) and (5) in the paper [1], the expression for Q is derived as (page 2223)

$$Q = -\int_{\lambda_{\min}}^{\lambda_{\max}} q(\lambda)\,\mathrm{d}N(\lambda)$$

$$= \int_{\lambda_{\min}}^{\lambda_{\max}} g_q \frac{\Delta P}{L_c(\lambda)} \frac{\lambda^4}{\mu} d_N \frac{d\left(\frac{\lambda}{\lambda_{\max}}\right)}{\left(\frac{\lambda}{\lambda_{\max}}\right)^{d_N+1}}$$

$$= \int_{\lambda_{\min}}^{\lambda_{\max}} g_q \frac{\Delta P}{L_c(\lambda)} \frac{\lambda_{\max}^{d_N}}{\mu} d_N \lambda^{3-d_N}\,\mathrm{d}\lambda.$$

With substitution of Eq. (2), we obtain

$$Q = \int_{\lambda_{\min}}^{\lambda_{\max}} g_q \frac{\Delta P}{L_0\left(\frac{L_0}{\lambda}\right)^{d_T-1}} \frac{\lambda_{\max}^{d_N}}{\mu} d_N \lambda^{3-d_N}\,\mathrm{d}\lambda$$

$$= g_q \frac{\Delta P}{\mu} \frac{\lambda_{\max}^{d_N}}{L_0^{d_T}} d_N \int_{\lambda_{\min}}^{\lambda_{\max}} \lambda^{d_T-1} \lambda^{3-d_N}\,\mathrm{d}\lambda.$$

Integrating and rearranging yields

$$Q = g_q \frac{\Delta P}{\mu} \frac{\lambda_{\max}^{d_T}}{L_0^{d_T}} \frac{\lambda_{\max}^{-(3+d_T-d_N)}\lambda_{\max}^3}{3+d_T-d_N} d_N \left(\lambda_{\max}^{3+d_T-d_N} - \lambda_{\min}^{3+d_T-d_N}\right).$$

According to the definition of $\beta = \lambda_{\max}/L_0$ given in the paper [1], the expression evolves to

$$Q = g_q \frac{\Delta P}{\mu} \beta^{d_T} \frac{\lambda_{\max}^3}{3+d_T-d_N} d_N \left(1 - \frac{\lambda_{\min}^{3+d_T-d_N}}{\lambda_{\max}^{3+d_T-d_N}}\right).$$

With the definition of $\alpha = \lambda_{\min}/\lambda_{\max}$ given in the paper [1],

$$Q = g_q \frac{\Delta P}{\mu} \lambda_{\max}^3 \beta^{d_T} d_N \frac{(1-\alpha^{3+d_T-d_N})}{3+d_T-d_N}. \tag{A1}$$

Eq. (A1) can be rewritten as

$$Q = g_q \frac{\Delta P A_0}{\mu L_0} \frac{L_0}{A_0} \lambda_{\max}^3 \beta^{d_T} d_N \frac{(1 - \alpha^{3+d_T-d_N})}{3 + d_T - d_N}.$$

Substituting $A_0 = L_0^2$ (page 2223) and $\beta = \lambda_{\max}/L_0$ yields

$$Q = g_q \frac{\Delta P A_0}{\mu L_0} \lambda_{\max}^2 \beta^{d_T+1} d_N \frac{(1 - \alpha^{3+d_T-d_N})}{3 + d_T - d_N}.$$

Using Darcy's law, the permeability equation is derived as

$$k = \frac{\mu L_0 Q}{\Delta P A_0} = g_q \lambda_{\max}^2 \beta^{d_T+1} d_N \frac{(1 - \alpha^{3+d_T-d_N})}{3 + d_T - d_N}. \quad \text{(A2)}$$

The expression of porosity, Eq. (C3), can be obtained by the similar procedures. It can be seen that Eqs. (C1)–(C3) are the mathematically integrated results.

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